CONCERNING THE ELASTIC ORTHOTROPIC MODEL APPLIED TO WOOD ELASTIC PROPERTIES

MODELO ELÁSTICO ORTOTRÓPICO APLICADO A LA MADERA

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ABSTRACT

Among the construction materials, wood reveals an orthotropic pattern because of unique characteristics in its internal structure with three axes of wood biological directions (longitudinal, tangential and radial). The effect of grain orientation on the elastic modulus constitutes the fundamental cause for wood anisotropy. It is responsible for the greatest changes in the values of the constitutive tensor components, and, thus in the elastic constant values of wood. The goal of this article is to verify the adequacy of the orthotropic model for wood, basically expressed by the modulus of elasticity E, related to a determined direction. Coordinate transformation between the materials axes (L, R and T) and Euler's angles is considered to use a constitutive equation for orthotropic materials. The main purpose of this analysis is to theoretically explore the coordinate transformation in the three - dimensional point of view and also to statistically compare the results of modulus of elasticity from compression test in a Brazilian wood species, *Guapuruvú (Schizolobium parahyba)*, with values obtained from a theoretical expression. The results from the analysis, in which the coefficient of determination (R-sq) was equal to 0.965 for a linear least squares analysis, showed that the orthotropic model is adequate to be applied.

Keywords: Modulus of Elasticity of Wood, Compression Test, Orthotropic Material.

RESUMEN

La madera, entre los materiales de construcción, presenta un comportamiento ortotrópico, esto debido a su estructura interna con tres ejes eldsticos de simetria: longitudinal, tangencial y radial. El efecto de la orientación angular de las fibras en el módulo eldstico constituye la causa fundamental de la anisotropia en la madera. Este efecto es responsable por los grandes cambios en los valores de los componentes del tensor constitutivo, y, consecuentemente en los valores de las constantes eldsticas de la madera. El objeto de este articulo es verificar la adecuación del modelo ortotrópico para la madera, expresado básicamente por el módulo de elasticidad E, referido a una dirección determinada. Se consider6 una transformación de coordenadas entre los ejes del material (L, R y T) y los ángulos de Euler para utilizar una ecuación constitutiva para materiales ortotrópicos. El propósito principal de este andlisis es explorar teóricamente la transformación de coordenadas, desde el punto de vista tridimensional y también, comparar estadisticamente los resultados del módulo de elasticidad obtenido a través del ensayo de compresión en la madera brasileña de la especie, *Guapuruvú,* con los valores obtenidos por medio de una expresión teórica. Los resultados de este andlisis, en el que el coeficiente de determinación (R-sq) fue igual a 0.965 en un andlisis de minimos cuadrados lineal, mostró que el modelo ortotrópico es válido para ser aplicado.

Palabras Claves: Módulo de Elasticidad de la Madera, Ensayo de Compresión, Material Ortrotópico.

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INTRODUCTION

The most general elastic constitutive model formulated to describe the mechanical behavior of material is the anisotropic model. This kind of model implies that there is no material symmetry, and mechanical properties in each direction are different. On the other hand, if there is material symmetry, the material can be denominated, for example, orthotropic or isotropic. In this context, the adequacy of a determined material for a certain elastic model is based on the existence of elastic symmetry axes. In these axes, denominated elastic principal axes, there is invariance of the constitutive relations under a group of transformations of coordinate axes.

In fact, the study of anisotropy is to develop the constitutive law that governs the elastic behavior of the material and, consequently, to determine the constitutive tensor, S_{ijkl}, and its components. In a completely elastic and anisotropic model this tensor has 81 unknown coefficients. By using adequate simplifications, this number can be reduced to 9 coefficients, which is denominated orthotropic model, or to 2 independent constants, which is the isotropic model.

Among the construction materials, wood follows an orthotropic pattern due to its unique internal structure along the three axes of wood biological directions (longitudinal, tangential and radial). Thus, there are 9 coefficients to be determined.

Because of the nature of wood, there are some parameters that can interfere with these elastic constants and such parameters include moisture content, specific gravity and the grain orientation.

On the other hand, focusing on the wood anisotropy, the variation of grain angle constitutes its fundamental cause. It is responsible for the greatest changes in the values of the constitutive tensor components, and, consequently in the wood elastic constants.

Many procedures have been used to analyze the wood behavior under a uniaxial direction and, determined the elastic constants of wood using uniaxial compression (or tensile) tests as well. Conversely, wood under a three-dimensional field view is less studied and consequently the constants of wood and their relationship are not fully quantified.

In this way, the aim of this paper is to verify the adequacy of the orthotropic model for wood, analyzing the results of compression tests in Brazilian wood species, *Guapuruvú (Schizolobium parahyba)* and then comparing them with the theoretical values obtained from a specific expression. In order to carry out this analysis, it is necessary to transform the angles measured on the surface of the specimen to Euler's angles and to apply the correct coordinate transformation tensor. Thus, this study addresses an investigation, on the basis of both theoretical and experimental aspects, in which it focuses on a three-dimensional coordinate transformation, which is usually less investigated in this area.

MATERIALS AND METHODS

Elastic Properties Of Anisotropic Materials

According to Love (1944) and Chen and Saleeb (1982), among others, the laws and equations that govern engineering problems are related to the stored energy in a solid. So, an elastic solid is

capable of storing the energy developed by the external work and transforms it into potential elastic energy that is denoted as strain energy. During this process, the body is deformed, but recovers its original shape and size once the external force is removed.

In this condition, if no energy is dissipated during the process of deformation, under adiabatic and isothermal conditions, the derived equations from this supposition are termed elastic models of Green and the material that makes the body as hyperelastic material. Thus, a hyperelastic material is the one that has a strain energy function, denoted by U.

The elastic material of Green is, in fact, a special case of the most general elastic material called elastic material of Cauchy, but considering the existence of the U , in order to maintain unaltered the laws of thermodynamics. These laws say that an elastic material produces no work in a closed loading cycle. Using the strain energy function and considering the Green elastic model, formulations of the constitutive laws for different classes of elastic materials can be established. So, consider a strain energy function given by:

$$U_0 = C_0 \delta_{ij} + \alpha_{ij} \varepsilon_{ij} + \beta_{ijkl} \varepsilon_{ij} \varepsilon_{kl} \tag{1}$$

where C_0 , δ_{ij} , β_{ijkl} α_{ij} are constants and ε_{kl} is the strain tensor. In view of the strain energy formulation where the strain energy has a stationary value in relation to the strain tensor, it is possible to set $C_0 = 0$. From equation (1) and considering that: $\sigma_{ij} = \dfrac{\partial U_0}{\partial \varepsilon_{ij}}$, the stresses can be expressed by:

$$\sigma_{ij} = \alpha_{ij} + (\beta_{ijkl} + \beta_{klij}) \varepsilon_{kl} \tag{2}$$

For an elastic body, the current state of stress depends only on the current state of strain. It may also be taken into the account the fact that $\alpha_{..} = 0$, since that the initial strain field corresponds to an initial stress free state, and $(\beta_{ijkl} + \beta_{klij})$ can be taken as C_{ijkl}, so, mathematically, the constitutive laws can be written as:

$$\sigma_{ij} = C_{ijkl} \varepsilon_{kl} \tag{3}$$

in which: σ_{ij} is the stress tensor, ε_{kl} is the strain tensor, C_{ijkl} is the tensor of material elastic constants.

Agreeing that $| C_{ijkl} | \neq 0$, the equation (3) can be expressed as:

$$\varepsilon_{ij} = S_{ijkl} \sigma_{kl} \tag{4}$$

where: S_{ijkl} is the compliance tensor.

The constitutive laws may also be written in matrix form as:

$$\{\sigma\} = [C]\{\varepsilon\} \tag{5}$$

Similarly for S_{ijkl}, we obtain:

$$\{\epsilon\} = \lfloor S \rfloor \{\sigma\} \tag{6}$$

On the other hand, according to Lekhnitskii (1981), all bodies, on the whole, can be divided into homogeneous and non-homogeneous bodies, and isotropic and anisotropic as well.

When a body is considered to be homogeneous, its physical properties, such as density-, remain invariant in all directions, in any of its points. For non-homogeneous body its properties are not constants.

If the elastic properties of the material are the same in certain directions, then the material exhibits symmetry with respect to these directions. If symmetry exists, the material is generally said to be isotropic. Otherwise, if there is no symmetry at all, the material is said to be anisotropic.

Another interesting issue to be pointed out is that when a body presents certain kinds of symmetry, the constitutive relations are simplified. These simplifications can be done in different ways just as those used by Love (1944), where the strain energy function remains unaltered by all symmetrical coordinate system substitutions. Thus, for example, a corresponding substitution given by three axes of elastic symmetry, $\chi'_i = -\chi_i (\chi' = -\chi; \gamma' = \gamma; z' = -z)$ does not change the value of U_0. Lekhnitskii (1981), on the other hand, performs these simplifications by in two different coordinate systems, symmetrical one to other. The author compared the obtained constitutive relations and identified the existence of the elastic symmetry.

A material with elastic symmetry under. the linear transformation $\chi'_i = l_{ij}\chi_j$, with l_{ij} being the coordinate transformation tensor, requires that the constitutive tensor, either C_{rspq} or S_{rspq}, comply with the following condition:

$$C'_{rspq} = l_{ri}\, l_{sj}\, l_{pk}\, l_{ql}\, C_{ijkl} \tag{7}$$

In this context, there are four cases of elastic symmetry, which are considered most important. They are: one plane of elastic symmetry, three planes of elastic symmetry (orthotropic material), transversely isotropy material and isotropic material. Since the purpose of this paper is to consider wood as an orthotropic body, we only analyzed this kind of elastic symmetry.

Thus, a body referred to a coordinate system χ is defined as orthotropic material if through each point there are three mutually perpendicular axes of elastic symmetry. Then, using the coordinate system χ_1, χ_2 and χ_3 (or χ, y, and z), perpendicular to the three planes of material symmetry and considering the elastic properties to be invariant under counterclockwise rotation 180" of about three axes, and using one at time as showed in Figure 1., it is possible to determine the constitutive tensor for orthotropic materials.
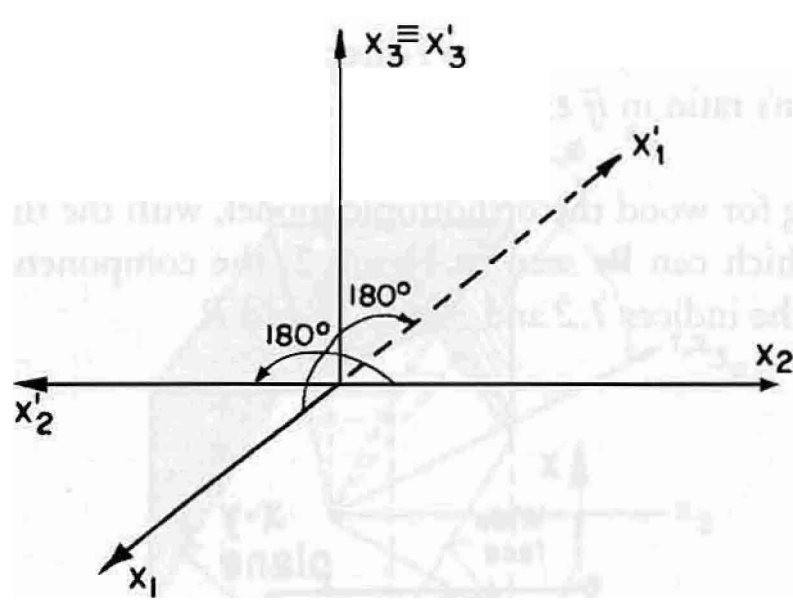
Figure 1-180° - Rotation about χ_3.

Consequently, we obtain **that:**

$$I_{ij} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \tag{8}$$

And we find, either C_{rspq} **or** S_{rspq} can be wtitten by:

$$S_{ijkl} = \begin{bmatrix} S_{1111} & S_{1122} & S_{1133} & 0 & 0 & 0 \\ S_{2211} & S_{2222} & S_{2233} & 0 & 0 & 0 \\ S_{3311} & S_{3322} & S_{3333} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{1212} & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{2323} & 0 \\ 0 & 0 & 0 & 0 & 0 & S_{3131} \end{bmatrix} \tag{9}$$

Now, **using** the engineering notation for elastic constants, **we** have that:

$$S_{ijkl} = \begin{bmatrix} \dfrac{1}{E_1} & -\dfrac{v_{21}}{E_2} & -\dfrac{v_{31}}{E_3} & 0 & 0 & 0 \\ -\dfrac{v_{12}}{E_1} & \dfrac{1}{E_2} & -\dfrac{v_{32}}{E_3} & 0 & 0 & 0 \\ -\dfrac{v_{13}}{E_1} & -\dfrac{v_{23}}{E_2} & \dfrac{1}{E_3} & 0 & 0 & 0 \\ & & & \dfrac{1}{G_{12}} & 0 & 0 \\ & Sim. & & & \dfrac{1}{G_{23}} & 0 \\ & & & & & \dfrac{1}{G_{31}} \end{bmatrix} \tag{10}$$

where: E_i is the modulus of elasticity related to i direction, G_{ij} is the shear modulus related to ij - plane and ν_{ij} is the Poisson's ratio in ij - plane.

In this way, arbitrating for wood the orthotropic model, with the three elastic principal axes denoted L, R and T, which can be seen in Figure 2, the components of the any tensor are determined by replacing the indices $1,2$ and 3 by L, T and R.
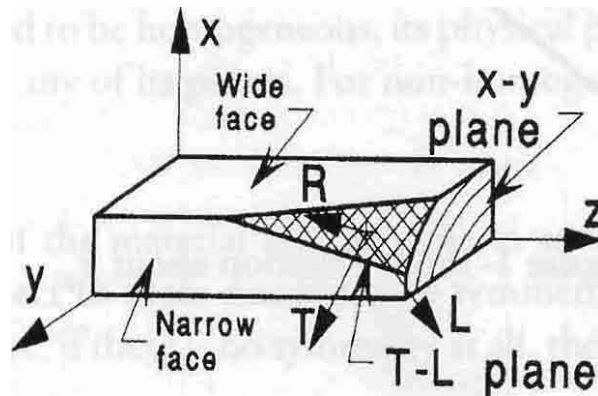

Figure 2- Material Axes and Board Axes for wood.

The effect of **Grain** Angle
In general, as already pointed out the variation of grain angle causes greatest changes in the constitutive tensor components, and, obviously in the wood elastic constant values.

Many researchers, in the theoretical and experimental point of view have long studied the effect of grain angle. One of the most important procedures was formulated by Hearmon (1948), who reported the effect of grain angles on all the components of S_{ijkl} showing that for wood it is possible to obtain negative values of Poisson's ratio, which emphasized the wood anisotropy.

Goodman and Bodig (1970), presented the following coordinate transformation matrix in order to determine the wood elastic properties with respect to rotation θ about the L axis and ϕ about R - axis:

$$\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi\cos\theta & \sin\phi\cos\theta \\ 0 & \cos\theta & \sin\theta \\ -\sin\phi & -\cos\phi\sin\theta & \cos\phi\cos\theta \end{bmatrix} \begin{Bmatrix} L \\ R \\ T \end{Bmatrix} \qquad (11)$$

The material axes and the board axes are: R, T, and L, χ_i (χ, γ, z), respectively. Figure 3 shows these axes.
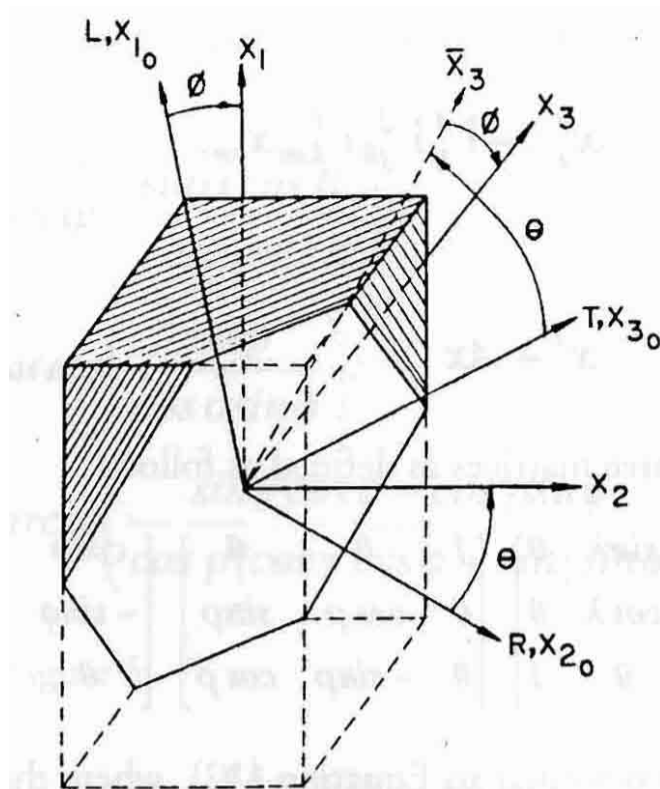
Figure **3-** Goodman and Bodig's coordinate transformation for wood.

Equation (11) can be generalized for two coordinate transformations that can be written by:

$$x_i' = |_{im} x_m = |_{ik}^2 |_{km}^1 x_m \tag{12}$$

In terms of tensor notation, $|_{ij}$ represent the set of direction cosines and I and 2 are, respectively, the first and the second rotations. Since no rotation about T was considered, this coordinate transformation is limited to cases where the L material axis lies in the χ_1- χ_3 plane.

Bindzi and Samson (1995) derived another coordinate transformation relation with rotation ϕ about L-axis and ψ about R-axis as follows:

$$\begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = \begin{bmatrix} cos\phi & -sin\phi cos\theta & sin\phi sin\psi \\ sin\phi & cos\phi cos\psi & -cos\phi sin\psi \\ 0 & sin\psi & cos\psi \end{bmatrix} \begin{Bmatrix} R \\ T \\ L \end{Bmatrix} \tag{13}$$

It can be noticed that the R-axis lies in the χ-γ plane. This equation can be got using Equation (12).

Both this and Goodman and Bodig's transformations are considered limited since it is not possible to obtain all relations between the board and material axes.

Hermanson (1996), studying the transformation of elastic properties for lumber to align these axes χ_i (χ, γ, z) with the material axes χ'_i (L, R, T), used three rotations λ, ρ and φ (denoted Euler 's angles) about χ, γ and z axes, as can be seen in Figure 4.

Thus, we can write that:

$$x'_i = | \, {}^3_{ij} | \, {}^2_{jk} | \, {}^1_{km} x_m \tag{14}$$

or in terms of matrix, that:

$$x' = Ax \tag{15}$$

with **A** being a product of three matrices as defined as follows:

$$A = \begin{bmatrix} \cos\lambda & \sin\lambda & 0 \\ -\sin\lambda & \cos\lambda & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos p & 0 \\ 0 & -\sin\rho & \sin\rho \\ & & \cos\rho \end{bmatrix} \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{16}$$

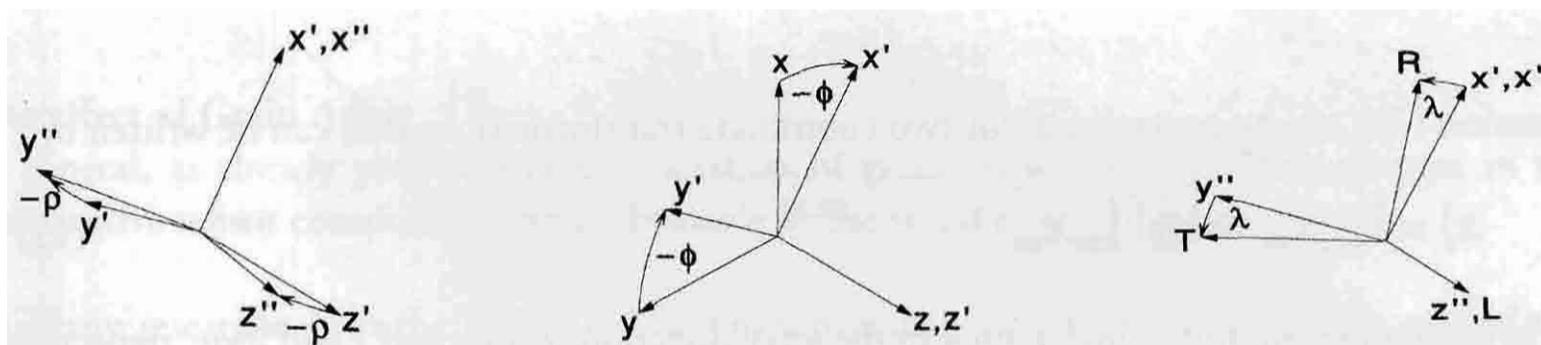Observe that Equation (14) is similar to Equation (12), where the superscript 3 represents the third rotation.

Figure 4 - Euler's angles λ, ρ and ϕ.

The final relation among these systems leads to:

$$\begin{Bmatrix} R \\ T \\ L \end{Bmatrix} = [A] \begin{Bmatrix} x \\ y \\ z \end{Bmatrix} \tag{17}$$

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$${}^0_R$$

$$\begin{Bmatrix} R \\ T \\ L \end{Bmatrix} = \begin{bmatrix} a_{RX} & a_{RY} & a_{RZ} \\ a_{TX} & a_{TY} & a_{TZ} \\ a_{LX} & a_{LY} & a_{LZ} \end{bmatrix} \begin{Bmatrix} x \\ y \\ z \end{Bmatrix} \tag{18}$$

where A is determined by:

$$A = \begin{bmatrix} \cos\lambda\cos\phi - \sin h\cos\rho\sin\phi & \cos\lambda\sin\phi + \sin\lambda\cos p\cos\phi & \sin h\,in\,p \\ -\sin\$\cos\phi - \cos\lambda\cos\rho\sin\phi & -\sin\lambda\sin\phi + \cos\lambda\cos p\cos\phi & \cosh i\,np \\ \sin\rho\sin\phi & -\sin p\cos\phi & \cos p \end{bmatrix} \tag{19}$$

After that, the three Euler's angles were related to the surface angles a, β and γ, through the following relations:

$$\phi = arctg\left(\frac{sina\,cos\,\beta}{-cos\,\alpha sin\beta}\right);$$

$$\rho = arctg\left(\frac{sinp}{cos\,\alpha sin\phi}\right);$$

(20)

$$\lambda = arctg\left(\frac{sin\gamma\,cos\phi - cos\gamma sin\phi}{cos\,\rho(cos\gamma\,cos\phi + sin\gamma sin\phi)}\right)$$
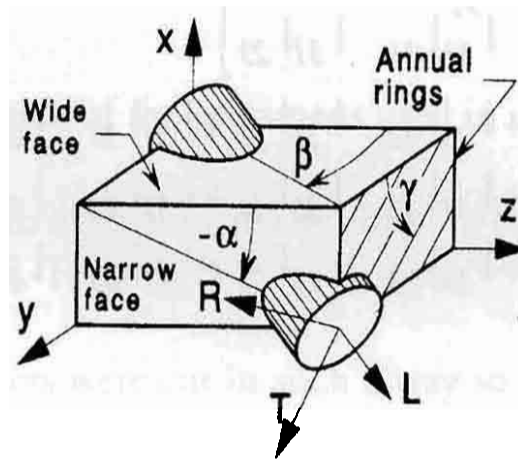
These angles can be seen in Figure 5:

Figure 5- Surface angles a , β and y.

In this way, it was possible to find the Euler's angles by knowing the surface angles and evaluating all wood elastic constants by using the complete coordinate transformation to elastic properties, described by Equation (7) or using Equation **(21)** given by:

$$S'_{rspq} = l_{ri}\,l_{sj}\,l_{pk}\,l_{ql}\,S_{ijkl}$$

(21)

or in terms of matrices by:

$$S' = K^T \cdot S \cdot K$$

(22)

where: l_{ij} K and $[K]^T$, the transpose of K, are function of components of the matrix A. The matrix K is the following matrix:

$$K = \begin{bmatrix} K_1 & 2K_2 \\ K_3 & K_4 \end{bmatrix}$$

(23)

and:

$$K_1 = \begin{bmatrix} l^2_{11} & l^2_{12} & l^2_{13} \\ l^2_{21} & l^2_{22} & l^2_{23} \\ l^2_{31} & l^2_{32} & l^2_{33} \end{bmatrix},$$ (24)

$$K_2 = \begin{bmatrix} l_{12}l_{13} & l_{13}l_{11} & l_{11}l_{12} \\ l_{22}l_{23} & l_{23}l_{21} & l_{21}l_{22} \\ l_{32}l_{33} & l_{33}l_{31} & l_{31}l_{32} \end{bmatrix},$$

$$K_3 = \begin{bmatrix} l_{21}l_{31} & l_{22}l_{32} & l_{23}l_{33} \\ l_{31}l_{11} & l_{32}l_{12} & l_{33}l_{13} \\ l_{32}l_{33} & l_{33}l_{31} & l_{31}l_{23} \end{bmatrix},$$

$$K_4 = \begin{bmatrix} l_{22}l_{33}+l_{23}l_{32} & l_{23}l_{31}+l_{21}l_{33} & l_{21}l_{32}+l_{22}l_{31} \\ l_{32}l_{13}+l_{33}l_{12} & l_{33}l_{11}+l_{31}l_{13} & l_{31}l_{12}+l_{32}l_{11} \\ l_{12}l_{23}+l_{13}l_{22} & l_{13}l_{21}+l_{11}l_{23} & l_{11}l_{22}+l_{12}l_{21} \end{bmatrix}$$

Now, from Equation (21), we can determine, for example, S_{1111}, S_{2222}, S_{3333} or, simply, the board elastic moduli [see Equation (10)], by:

$$\frac{1}{E_i} = \frac{a^4_{Ri}}{E_R} + \frac{a^4_{Ti} - 2a^2_{Ri}a^2_{Ti}\nu_{TR}}{E_T} + \frac{a^4_{Li} - 2a^2_{Ri}a^2_{Li}\nu_{LR} - 2a^2_{Ti}a^2_{Li}\nu_{LT}}{E_L} + \frac{a^2_{Ti}a^2_{Li}}{G_{TL}} + \frac{a^2_{Li}a^2_{Ri}}{G_{LR}} + \frac{a^2_{Ri}a^2_{Ti}}{G_{RT}}$$ (25)

where: $i = x, y, z$.

It is noted that the terms: a_{Ri}, a_{Ti}, a_{Li} are obtained from Equations (18) and (19).

Experimental Methods

The experimental data used to analyze the adequacy of the wood elastic behavior for orthotropic elastic model were obtained in Lamem- São Carlos-Brazil, following the procedure used by Mascia (1993). The specimens which were used consisted of wood blocks of 6 cm x 6 cm x 18cm of *Guapuruvú (Schizolobium parahyba)* species obtained from tree stem, according to the following schematic. The moisture content was around 12% at ambient temperature (25" C) and the specific gravity around *0.40* g/cm³.
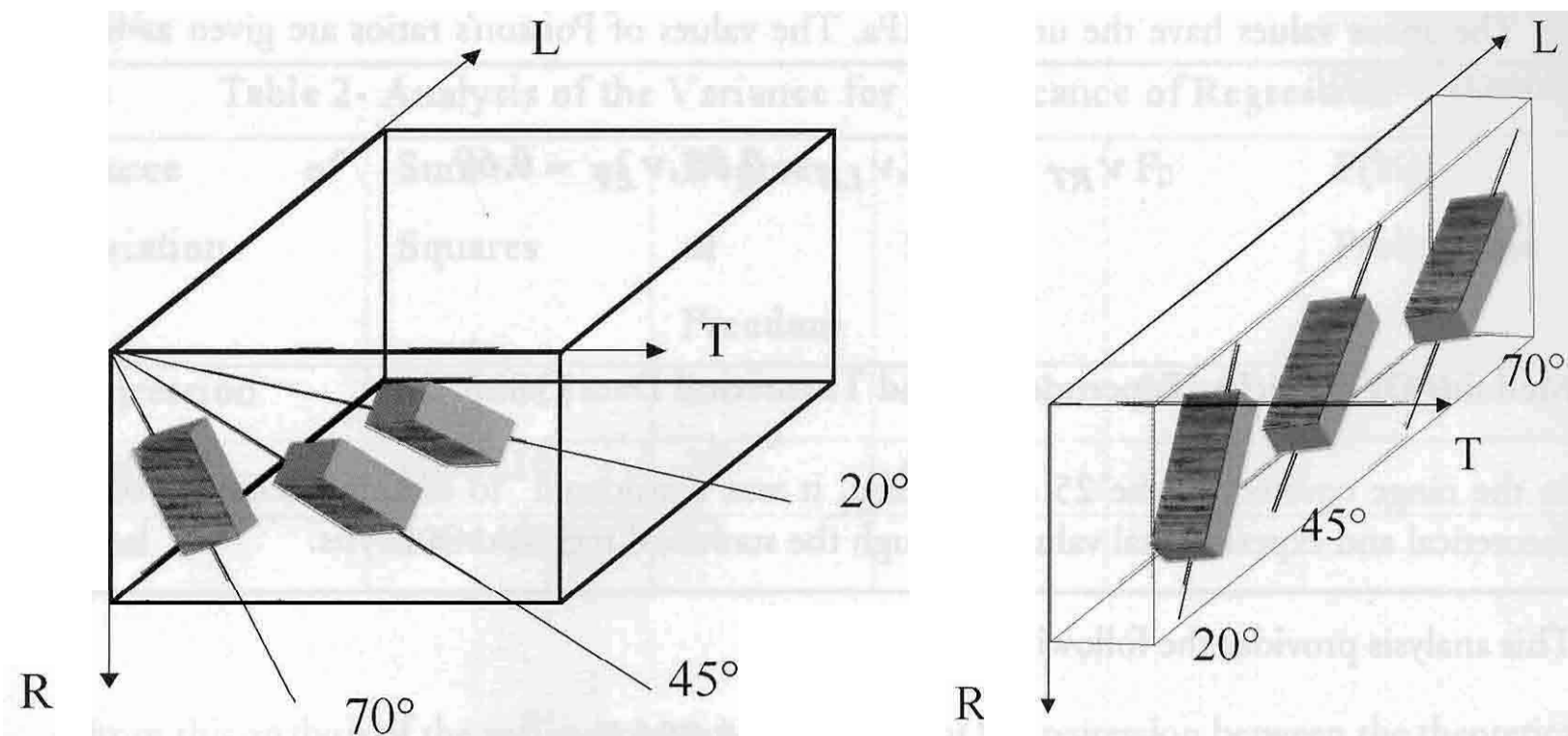
Figure 6- Scheme of the specimens used in compression test

The goal of this procedure is to determine the modulus of elasticity in some grain orientations determined with respect to the R-T, the R-L and T-L plane.

To achieve this, firstly, lumbers were cut in such a way so that one axe direction in the R-T plane varying in the following angles: 0°,20°,45°,70° and 90° and the length axe being parallel to the L direction. After this, blocks were obtained from these lumbers following the same direction but varying the angles over the lumber axe in the R-T plane by 0°, 3°,5°,7° and 10″ . In this way, 25 specimens were obtained for the compression tests. We have to emphasize that this procedure was very carefully designed and needs great care to accomplish.

It was used the AMSLER test machine with 250 kN load capacity and strain gages to measure the strains.

RESULTS AND DISCUSSIONS

In this work, only the modulus of elasticity in z-axis was investigated by comparing the experimental data, denominated E_{exp} with and the theoretical predictions from Equation (25), E_{theo}. To use Equation (25), it is necessary to know the following elastic constants:

$$E_R, E_T, E_L, G_{TL}, G_{RT}, G_{LR}, \nu_{RT}, \nu_{LR}, \nu_{LT}$$

These parameters were determined from the compression test data [Mascia (1993)], whose values are :

$$E_R = 519, E_T = 287, E_L = 3507, G_{TL} = 421, G_{RT} = 73, G_{LR} = 378,$$

The above values have the unit of MPa. The values of Poisson's ratios are given as follows [Mascia (1993)]:

$$v_{RT} = 0.83, v_{LR} = 0.56, v_{LT} = 0.69$$

Modulus Of Elasticity: Experimental **And** Theoretical Data Comparison

In the range covered by the 25 sets of data, it was considered to establish relations between theoretical and experimental values, through the statistical regression analysis.

This analysis provides the following regression:

$$E_{theo} = 248 + 0.896\, E_{exp} \tag{26}$$

with : R-sq = 96.5% (Coefficient of Determination) and where E_{theo} is the modulus of elasticity in z-axis, E_z , calculated from Equation (25) and E_{exp} is obtained from the experimental data.

Detailed data both for the theoretical calculations and experimental measurements are given in Table 1 whereas Table 2 shows the Analysis of Variance. The analysis was based on the methods of Montgomery and Peck (1992) and Ryan (1994), and Minitab software.

Table 1

Table 1 Modulus of Elasticity E_i (MPa)					
Specimen	E_{exp} (MPa)	E_{theo}	**Specimen**	E_{exp}	E_{theo}
0 – 0 – 0	3500	3495	45 –70	311	494
0 – 20	1884	2102	45 – 90	250	519
0 – 45	876	694	70 –0– 7	3391	3245
0 –70	413	341	70 – 20	1500	1963
0 – 90	519	519	70 –45	656	858
20 – 0 – 3	3666	3442	70 – 70	379	561
20 – 20	1489	2045	70 –90	211	519
20 – 45	489	674	90 – 0–10	3500	3112
20 – 70	379	363	90 – 20	1400	1900
20 – 90	338	519	90 – 45	677	864
45 – 0 –5	3700	3353	90 – 70	486	572
45 – 20	1692	1987	90 - 90	287	519
45 – 45	588	746			

Table *2*

Table 2- Analysis of the Variance for Significance of Regression					
Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0	P(%) Probability
Regression	29810815	1	2981081	663.61	0.0001
Residual	1033210	23	44922		
Total	30844025	24			

From this analysis of the variance we can conclude that the regression between the theoretical values and the experimental data is significant. This means that we can reject the null hypothesis with a high level of significance. We can observe that if the observed value of F_0 is large, the parameter E_{exp} is non- zero for a reached probability p . In other words, the agreement among the theoretical values of the elastic modulus and the experimental values described by Equation (26) is satisfactory.

Figure 7 shows the regression plot, in which, by analyzing the prediction interval, with 0.95 confidence coefficient, we can observe that some results in the 25-data set did not adequately fit in the confidential interval of the linear model but all of them fitted adequately in the predicted interval. This statistically reflects the capacity of predicting of this model.

Regression Plot

$Etheo= 248.205 + 0.896255 Eexp$
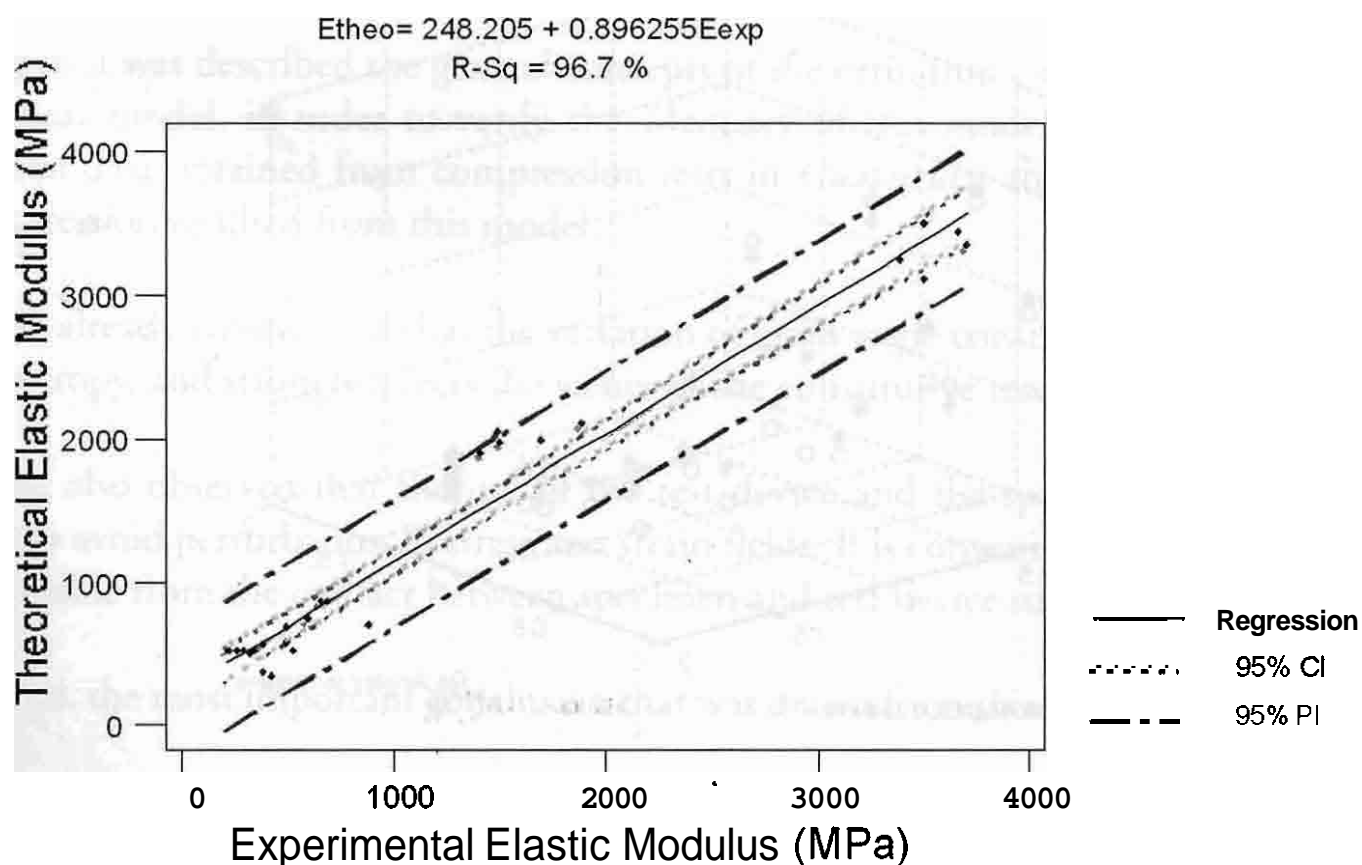R-$Sq = 96.7$ %

Figure 7- Regression Plot Of Experimental and Theoretical Modulus of Elasticity (In MPa)

To better illustrate this argument we present Figure 8 and Figure 9 showing the agreement between the experimental and theoretical results as a function of grain angles. To construct these three-dimension diagrams of Elastic Modulus and Euler's angles, and the lateral view of this figure, Figure 10, it was used the Matlab software and the reference: Hanselman and Littlefield (1998).
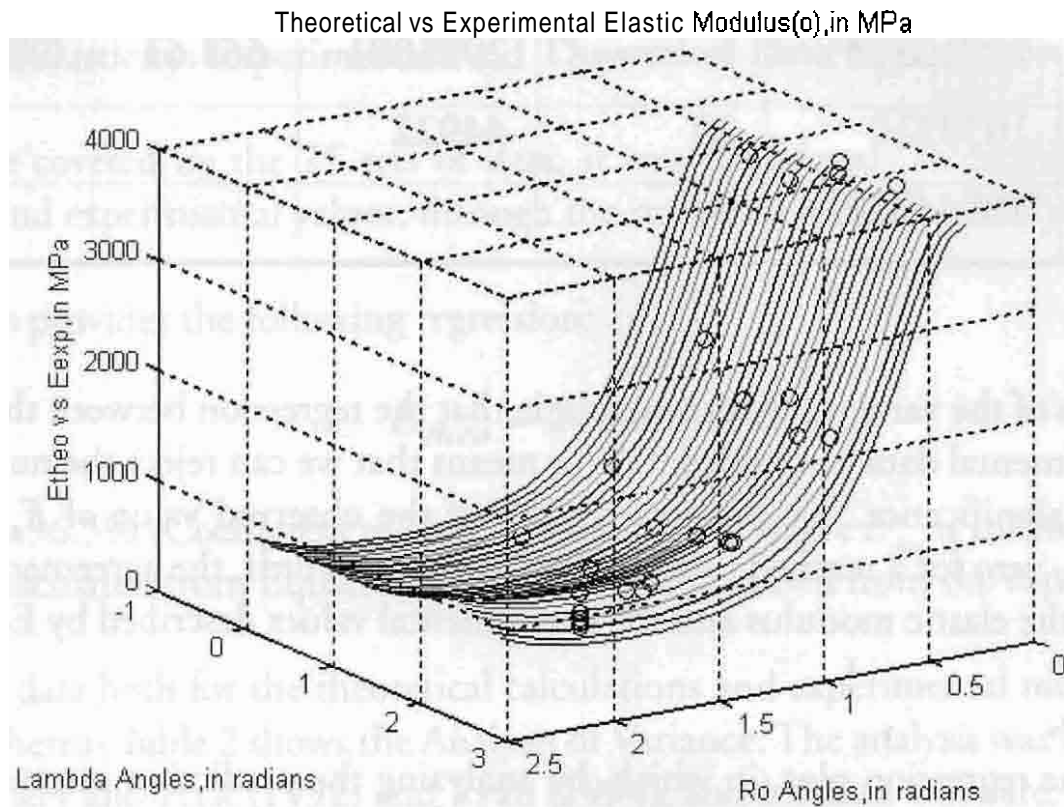
Figure 8- Theoretical Curve and Experimental Values of the Modulus of Elasticity.
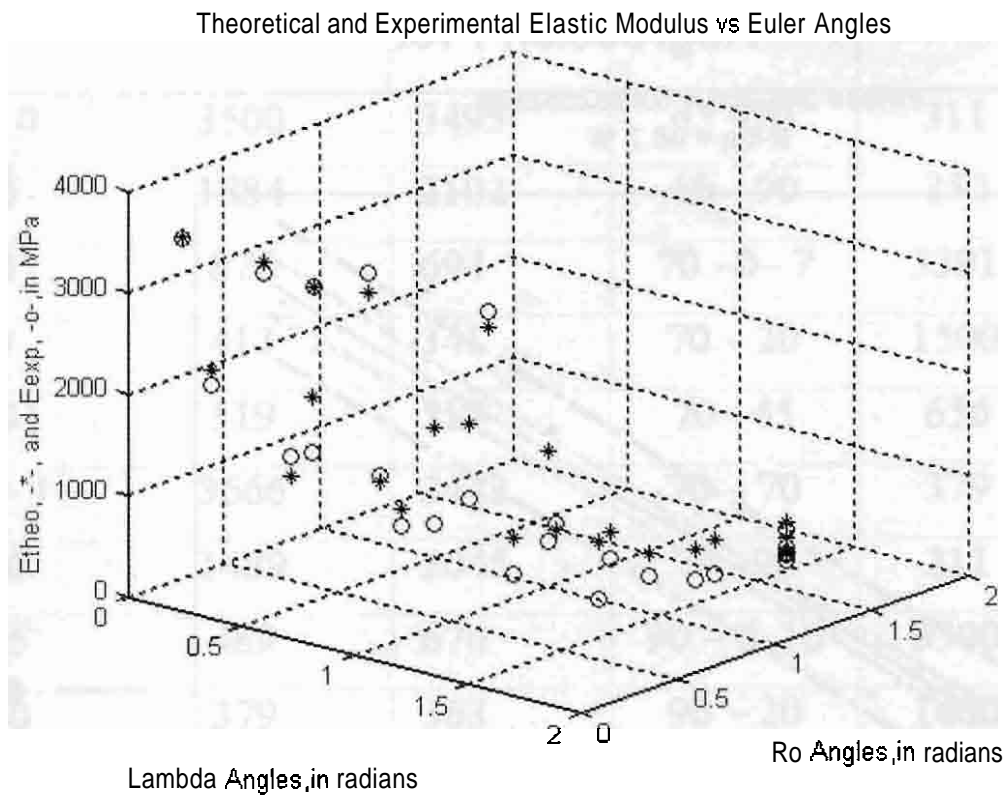
Figure 9- Theoretical and Experimental Values of the Modulus of Elasticity.
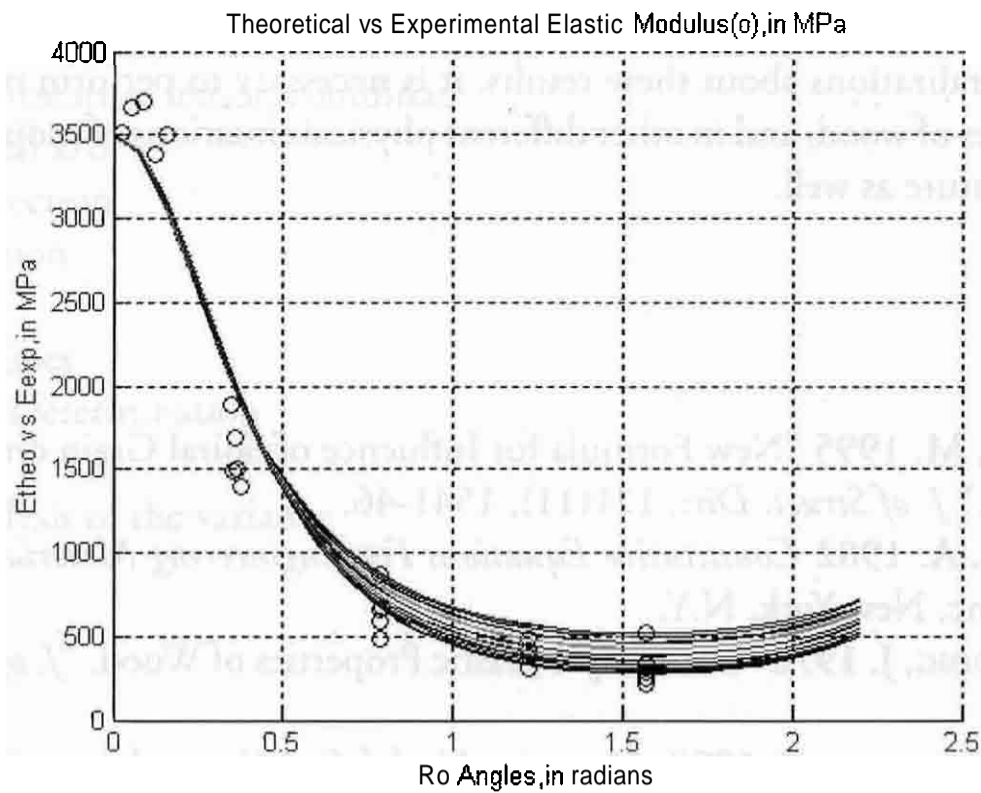
Figure 10- Lateral View Of The Three -Dimension Diagram Of Modulus of elasticity From The Theoretical And Experimental Data.

We observe that the continual curve is the theoretical curve based on the expression of Equation (25) and the plotted points are the experimental data.

In summary, we can point out that there are some values that perturb the linear model, but, in general, the regression analysis provides results that are considered satisfactory for wood.

CONCLUSIONS

In this paper, it was described the general concepts of the orthotropic elastic model, particularly the rectilinear model, in order to verify the adequacy of this model for wood, by analyzing experimental data obtained from compression tests in *Guapuruvú* and theoretical data from a specific expression resulted from this model.

We have already commented that the variation of grain angle constitutes the main reason,for wood anisotropy, and strongly affects the values of the constitutive tensor components.

We have also observed that the use of the test device and the specimen configuration are important to avoid perturbation in stress and strain fields. It is convenient to measure the strains as far as possible from the contact between specimen and test device surface.

In general, the most important conclusion that was drawn from this study can be summarized as follows:

• The agreement between the rectilinear orthotropic model, described by the theoretical values and the experimental values, can be considered satisfactory. The present statistical analyses indicated that only some results of the data did not adequately fit in the model especially because wood to be a non-homogeneous and an anisotropic material.

It is important to notice that this conclusion is restricted to the current experimental data. In order to make generalizations about these results, it is necessary to perform more tests taking into account other species of wood, and in other different physical situations of moisture content, specific gravity and temperature as well.

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NOTATION

C_0, δ_{ij}, β_{ijkl}, α_{ij}: constants

i,j,k: indices

ε_{kl}: strain tensor, strain

σ_{ij}: stress tensor, stress

x_i, x_1, x_2 and x_3 (or x ,y, and z): coordinate system

U_0: strain energy function

C_{ijkl}: tensor of material elastic constants

S_{ijkl}: compliance tensor

E_i: modulus of elasticity related to i direction

G_{ij}: shear modulus related to ij -plane

v_{ij}: Poisson's ratio in ij - plane

E_{exp}: modulus of elasticity from the experimental data

E_{theo}: modulus of elasticity in z-axis from theoretical data

A, K: matrix,

α_{ij}: elements of tensor or matrix

l_{ij}: coordinate transformation tensor, coordinate

x'_i (L, R, T): material axes

L: Longitudinal direction

T: Tangential l direction

R: Radial direction

A, p , φ: Euler 's angles

R-sq: Coefficient of Determination

p: probability

$F_{,,}$: parameter of analysis of the variance