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ARTÍCULO

Assessing the effectiveness of static heuristics for scheduling lumber orders in the sawmilling production process

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Abstract:
Although optimization models can be used to plan the production process, in most cases static heuristics, such as earliest due date (E), longest processing time (L), and shortest processing time (S), are used because of their simplicity. This study aims to analyze the production cost of the static heuristics and to determine how this cost relates to the size of the production orders in the sawmilling industry. We set a planning problem with different orders and due dates and solved it using two cost-minimization models to compare their solutions. The first was a planning model (PL) where orders were split up into products demand by period, and the second, a planning scheduling (PS) where the sequence of processing orders based on static heuristics was assumed as known. In the latter, the minimum production cost for each static heuristic was found. In both models, the same resource constraints were assumed. The costs showed no significant changes based on order sizes. However, 0,5 % of orders were delayed using PS-E, and 17 % of orders were delayed using PL. PL was an efficient solution method when changing the orders´ size and when looking for the best static heuristic to process the orders. However, PS-E showed the ability to reduce the backlog close to zero while the PL backlog ratio was 17 %. No penalties were applied to backlogs due to their subjective nature; however, when shortages occurred, the demand was unmet or backlogged with substantial costs. Thus, in case the proposed method is adopted using a conservative backlog cost, a sawmill producing under the cut-to-order environment that produces 300000 m^3 /year would reduce backlogged orders by 51000 m^3. If the holding lumber cost is 2 \$/m^3, annual savings would be \$408000.

Keywords: Lumber planning, lumber scheduling, backorders cost, static heuristics, minimum production costs, sawmills.

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Introduction

In the year 2020, the forest industry emerged as one of the key pillars of the Chilean economy, contributing 2,1% to the national GDP. Forty percent of all industrial roundwood was used in sawmills, with a similar percentage allocated to pulpwood and paper production. The remaining portion was used in the manufacturing of lumber, panels, and other products, many of which were exported. Chilean forest companies exhibit a high level of integration, encompassing operations in forestry, cellulose, paper, lumber, plywood, and energy production. Nevertheless, there has been a growing focus on lumber production, driven by the need to address significant efficiency gaps.

In most large sawmills in Chile, the planner faces the task of balancing the utilization of logs. This involves assigning sawing patterns to logs to meet orders deadlines while ensuring a certain performance threshold regardless of its impact on production costs. Unfortunately, these performance thresholds often clash with the efficient utilization of capacity. For instance, this conflict arises when scheduling small batches for numerous orders without considering setup costs. Conversely, scheduling large batches for a few orders prioritizes maximum productivity. This paper seeks to quantify the impact of static heuristics for scheduling, widely employed by sawmill planners, and the influence of lumber batch size on production costs. The objective is to identify the production setting in which static heuristics outperform others. This section begins with a literature review, highlighting gaps in existing research, and establishes the research objectives.

Lumber planning is based on a balance between timber supply and lumber demand. Sawmills that process timber from natural forests typically employ the cut-to-stock approach, whereas those processing timber from plantations favor the cut-to-order approach (Vanzetti *et al.* 2018, Marier *et al.* 2014). The Operational Lumber Planning Problem (OLPP) can be categorized as both a lot sizing and scheduling problem, determining when orders should be fulfilled while taking capacity into account to minimize backlog and inventory costs.

The OLPP has been approached through Mixed Integer Programming (MIP) formulations, heuristics methods, and hybrid approaches that combine the strengths of both. MIP models play a crucial role in minimizing the costs associated with raw materials, inventory, and backlog while adhering to available capacity constraints (Clark 2003). However, scheduling presents challenges due to sequence-dependent setups, where each machine's setup time for a task is influenced by both the current job and the previous job for which the machine is setup. Additionally, multiple setups occur within a planning period. As a result, the MIP model for lot sizing and scheduling involves numerous binary variables, leading to computational intractability. While exact formulations produce high-quality solutions, they often come with impractical solution times. To overcome this complexity, a strategy involves relaxing the binary variables and constraints, breaking down the problem into smaller components to determine lot size, and establishing a sequence of lot setups (de Araujo *et al*. 2007 and Clark 2003).

Maness and Adams 1991, followed by Maness and Norton 2002, conducted early research on lumber planning and scheduling. The MIP model they developed aimed to maximize revenues by generating a bucking-sawing pattern and selecting the mix of logs to meet sales demand. This approach is suitable when backorders and inventories are allowed.

Sawmills commonly use MIP models with a focus on either volume or value. However, if dynamic prices can be applied and updated during sawing operations, optimized plans can yield more accurate plans resulting in using fewer logs (Todoroki and Ronnqvist 2010).

Adopting a cut-to-order environment, where logs are sorted by diameter and processed in batches, has improved the results of the OLPP through timber transfer decisions among plants within a supply chain context (Singer and Donoso 2007). A heuristic approach mimicking planner schedulers and a MIP model were developed and benchmarked by Huka and Gronalt 2017. The MIP model outperforms the heuristics, however, when applying the MIP model in a rolling planning horizon, the best heuristic outperforms it owing to the end effects included in the model.

An alternative approach was undertaken by Dumetz *et al.* 2019. A simulation planning method simulated the arrival of new orders, demand, capacity, inventories, rolling planning horizon, and coordination mechanisms, resulting in an increased service level compared to centralized planning methods.

Furthermore, Vergara *et al.* 2015 pointed out that the lumber planning problem has been tackled with conflicting objectives, where a solution optimized for one objective adversely affects another. To address this conflict, Broz *et al.* 2019 applied a Goal Programming formulation, balancing problem metrics to showcase the importance of reconciling decision-makers' local objectives. Additionally, a model feature facilitating the planning of log transportation to the sawmill was introduced by Vanzetti *et al.* 2018. However, realistic modeling formulation in the cut-to-order environment discussed would have required the inclusion of slack variables and penalties.

In a cut-to-stock environment, Wery *et al.* 2014 emphasized the challenges that arise when dealing with customized orders. They proposed a simulation-optimization sequential

framework to evaluate how mill settings and the introduction of new log classes impact the sawmill products mix when customized products are added to the plan during the planning horizon. Furthermore, Ben Ali *et al.* 2019 outlined the advantages of integrating sales and operations planning and order promising based on revenue management concepts. Their MIP model determines sales decisions that maximizes profit while enhancing service levels for high-priority customer orders.

In a cut-to-order environment, incorporating remanufacturing operations, Vanzetti *et al.* 2019 developed a MIP model for day-ahead scheduling optimization. This model utilizes both self-generated and data-based cutting patterns, emphasizing the advantage of generating all possible cutting patterns, which surpasses previous research efforts.

To address the OLPP in a more general context, Kaltenbrunner *et al.* 2020 integrated existing approaches and evolving scenarios. They developed a modular, flexible, and generic planning method, with MIP model cases differing in demand fulfillment constraints, optimal use of raw material constraints, and the division of the planning horizon.

While previous reviews have focused on MIP and heuristic methods, this research aims to quantify the impact of static scheduling heuristics on production costs. Therefore, a subsequent review of scheduling methods applicable to this initiative follows.

Scheduling is typically conducted by shift leaders without considering cost consequences. A schedule arranges jobs in a specific order and determines their start and completion times (Maccarthy and Liu 1993). Operations Research (OR) methods can be employed to address scheduling problems, aligning with the production flow and production system. Different objective functions, indirect objectives, and metrics can be produced based on the problem scale, allowing for the determination of solution technique performance gaps (for further details, refer to Maccarthy and Liu 1993). While the optimal schedule serves as a

performance measure, it is important to note that costs and profits cannot be directly linked to schedules. Indirect objectives, such as completion time, flow time, lateness, and tardiness are used instead (Sipper and Bulfin 1997).

OR methods employed to tackle scheduling problems encompass optimal methods, enumerative optimal MIP methods, and heuristic methods, the latter involving a certain degree of closeness to polynomial time (Maccarthy and Liu 1993). Additionally, static heuristics offers optimal solutions for certain problems, and they are a function of the order data (Possani 2001). In fact, static heuristics prioritize jobs and schedules them based on a rule that recognizes the priorities of all the orders that are waiting to be processed on a machine. The Shortest Processing Time (SPT) static heuristic prioritizes jobs based on their processing times, while Earliest Due Dates (EDD) static heuristic prioritizes jobs based on their due dates. The Longest Processing Times (LPT) static heuristic organizes jobs in a sequence with non-increasing processing times. Unfortunately, neither SPT nor LPT considers due dates, as these static heuristics primarily focus on minimizing flow time (i.e., cost minimization). If customer satisfaction is a crucial factor, EDD is the preferred choice because it minimizes tardiness (Sipper and Bulfin 1997).

Yaghubian *et al.* 2001 developed a model to address a dry kiln scheduling problem, a version of scheduling “n” jobs on “m” parallel machines. Jobs were assigned to kilns in accordance with their due dates, and the optimization problem revolved around allocating jobs to kilns. Each job was associated with demands (e.g., orders), and the due date was ensured by imposing a constraint that guaranteed the fulfillment of the last job on demand by the due date.

Furthermore, Maturana *et al.* 2010 tackled the OLPP using a Linear Programming (LP) model and a heuristic approach that emulated planner decisions. They introduced log supply

and demand perturbations. While the LP model proved to be faster than the heuristic approach for most scenario perturbations, it lacked explicit scheduling rules present in the heuristic method, such as LPT. In a similar vein, Pradenas *et al.* 2004 used a MIP model, benchmarking results with an artificial intelligence heuristic procedure based on a Tabu search algorithm, demonstrating promising performance when compared with an exact problem solution.

Meanwhile, Marier *et al.* 2014 formulated MIP models to address the OLPP. The MIP model for scheduling batches of lumber was simplified by imposing the LPT sequence to process orders on machines rather than generating and testing many combinations of order schedules. This simplification aimed to reduce complexity and make the problem more tractable (Vanzetti *et al.* 2019).

Although efforts have been made to address the OLPP, much of the research has prioritized the due date of orders over production cost. Limited attention has been given to understanding how static heuristics for scheduling can impact costs. This research quantifies the effect on cost when using static heuristics for scheduling lumber orders on sawmills.

The following sections provide a methodology outlining the approach, model formulations, data utilized, and the comparison strategy. Subsequently, the results are presented and benchmarked against previous research, followed by a detailed discussion of findings and research limitations. We conclude with summaries of conclusions, implications, and study limitations.

Materials and methods

The methodology was designed to examine the impact of incorporating static heuristics, such as EDD, SPT, or LPT, in processing lumber production orders on cost, backlog, and overdue orders while solving the OLPP. The formulated models designed to address this problem are described, along with an explanation of how the demand for lumber products was utilized to generate various lumber demand scenarios. The subsequent section details the procedure for executing the models and elucidates how the problem data were input to explore variations in the problem.

Problem setting

The OLPP was configured with 10 log diameters, a library of 100 sawing patterns, sawmilling and anti-stain treatment processes, and 10 anti-stain lumber. The OLPP was approached using a lumber planning methodology, which is comparable to a lumber scheduling approach. For lumber planning, a timeframe of six periods, corresponding to weekdays, was employed. In contrast, the lumber scheduling approach utilized a period of 138 hours, equivalent to 6 days of 23 hours each. Each order consisted of a set of 10 lumber products, and the volume of every order matched the demand for the lumber products that needed to be fulfilled within a specific period for the lumber planning problem. This volume

equivalency was translated into an order with a due date specified in hours to be satisfied within the scheduling problem.

Lumber manufacturing data

The data set utilized in this study was sourced from a lumber manufacturing company in southern Chile. To uphold company confidentiality, certain aspects such as logs, labor, and inventory costs were adjusted, although they closely approximate actual figures. Log supply was assumed to be unlimited to prevent infeasibilities in the model due to log availability, and to maintain a focus on the impact of static heuristics rather than other factors (Singer and Donoso 2007, Maturana *et al.* 2010).

An anti-stain process is incorporated into the lumber production of Radiata pine due to high proportion of sapwood, aiming to prevent fungal activity. Hence, sawing and anti-stain processes were considered as operations in this research.

The initial demand for lumber products was defined as the average production that the library of sawing patterns could produce when utilizing the entire sawmill capacity. The demand was based on a sawmill with a capacity of consuming up to 111 m^3 of logs per hour and producing up to 62 m^3 per hour of lumber. These figures varied based on log distributions and the sawing pattern library. This assumption aligns with the current working conditions of the company and is consistent with the case study by Maturana *et al.* 2010. To assess the impact of static heuristic (sequences) on the OLPP, five lumber product demand scenarios

were generated based on the initial lumber product demand. These scenarios focused on varying rates applied to the initial demand between periods and among lumber products, with thirty values generated for each scenario (Table 1).

Table 1: Scenarios of lumber products demand.

Scenarios	Description	Rate of variation
Large (La)	Strong Market	The initial demand for lumber products was randomly adjusted, either increased or decreased, by 3%.
Small (Sm)	Weak Market	The initial demand for lumber products was first reduced by 20% and then randomly adjusted, either increased or decreased, by 3%.
Mixed sizes (Mi)	Mixed strong and weak market	This demand can assume either a large or a small value for lumber products. The data generation involved selecting between a previously generated large or a small demand value for lumber products.
High rate of Variation (Hi)	High volatility in market	The original demand for lumber products was adjusted, either increased or decreased, by 50%.
Low rate of Variation (Lo)	Low volatility in market	The original demand for lumber products was adjusted, either increased or decreased, by 20%.

These five scenarios, each comprising 30 sets of lumber product demands organized by period or order, were fed into the models described in the next section, leading to a total of 600 model runs.

Decision-making model formulations

Two planning and scheduling models, namely a multi-period planning model (PL) and a planning-scheduler model (PS) with 3 versions, were developed and performances were compared. The initial optimization model (PL) is a MIP model (PL) that was transformed into a single-period allocation of orders problem, referred to as planning-scheduler (PS). The PS includes three versions: a scheduling model with Earliest Due Date static heuristic (PS-E), a scheduling model with Longest Processing Time static heuristic (PS-L), and a scheduling model with Shortest Processing Time static heuristic (PS-S). In the E static heuristic, orders are scheduled in ascending order of due dates. The S static heuristic involves sorting orders by increasing processing times. The L static heuristic schedules orders by decreasing processing times (Table 2).

Table 2: Models' description.

Model name	Model focus
PL	The multiperiod production planning model allocates the production orders over the planning horizon period during which customers require product delivery. This model was selected as a benchmark solution, as it represents the classic approach to lumber production planning.
PS-E	The PS-E optimizes the resources by adhering to the Earliest Due Date (E) static heuristic, wherein production orders are scheduled in ascending order based on their due dates.
PS-S	The PS-S optimizes the resources by adhering to the Shortest Processing Time (S) static heuristic, wherein production orders are scheduled in ascending order based on their processing times.
PS-L	The PS-L optimizes the resources by adhering to the Longest Processing Time (L) static heuristic, wherein production orders are scheduled in descending order based on their processing times.

In the PL model, the demand for lumber product must be fulfilled within a specific period of the planning horizon (i.e., days of the week). Although the formulation did not explicitly include due dates, demand was allocated to certain periods. Thus, the model implicitly ensured demand satisfaction within a period, equivalent to fulfilling an order containing a set of lumber product demands for a specific a due date (Equation 1 and Equation 2).

$$V\_at_{os} + IAT_{os-1} = D\_at_{os} \quad \forall o, s = 1 \qquad (1)$$

$$V\_at_{os} + IAT_{os-1} - IAT_{os} + B\_at_{os} = D\_at_{os} \quad \forall o, s > 1 \quad (2)$$

Where:

o: Lumber product

s: Planning periods

V_at_{os} :Anti-stain treatment production (m^3) of lumber product o in period s.

B_at_{os} :Backlog volume (m^3) of anti-stain treated lumber products o in period s.

D_at_{os} :Demand for anti-stain lumber (m^3) product o in period s.

IAT_{os} : Anti-stained lumber inventory (m^3) of product o in period s.

$$\sum_{ijkp} U_{i,j,k,p} \times Prod\_sw_i + A1_p - B1_p - d\_at_p \leq 0 \quad \forall p,1=1...p \quad (3)$$

Where:

i: Diameter,

p: Orders of lumber products, 1…m

o: Lumber, 1…n

U_{ijkp}:Vol. of logs (m^3) of diameter i, log grade j, sawn with sawing pattern k, to satisfy order p.

$Prod\_sw_i$: Sawmilling productivity (h/m^3) when sawing log diameter i.

$A1_p$: Advanced time (h) of sawn lumber order p.

$B1_p$: Backlogged time (h) of sawn lumber order p.

d_at_p : Due date (h) for order p.

Therefore, two models were formulated to address the same OLPP problem from different perspectives (Figure 1). Initially, the PL model met lumber product demand per period, implicitly ensuring the due dates of the set of lumber orders constituting that demand. In each period, PL generated lumber to fulfill demand until reaching production capacity. Simultaneously, it determined the volume of raw materials and products for inventory. While this approach ensures timely deliveries to customers, it is costly due to its underutilization of mill capacity, disregarding setup costs and the number of production batches needed to satisfy orders (Wery *et al.* 2014). Conversely, the PS model was formulated, wherein lumber product demands are consolidated into orders with specified due dates processed in a sequence. Orders are applied in a sequence based on either processing times or due dates, as suggested by Yaghubian *et al.* 2001. The periods of the formulation were dropped off, then an "earliness" constraint was added that ensured that the processing time of order p, and processing times of preceding times orders must be less than or equal to the due date of order p (Equation 3). Accordingly, this constraint compelled orders to be processed without violating their due dates, however, due to model infeasibilities, the earliness constraint was relaxed, allowing overdue orders without penalties.

The main difference between PS and PL lies in the fact that PS operates on a one-week equivalent period, whereas PL involves six periods, corresponding to days of the week. Additionally, lumber products demand by period in PL are associated with orders containing a set of lumber that must be satisfied by a specific due date.

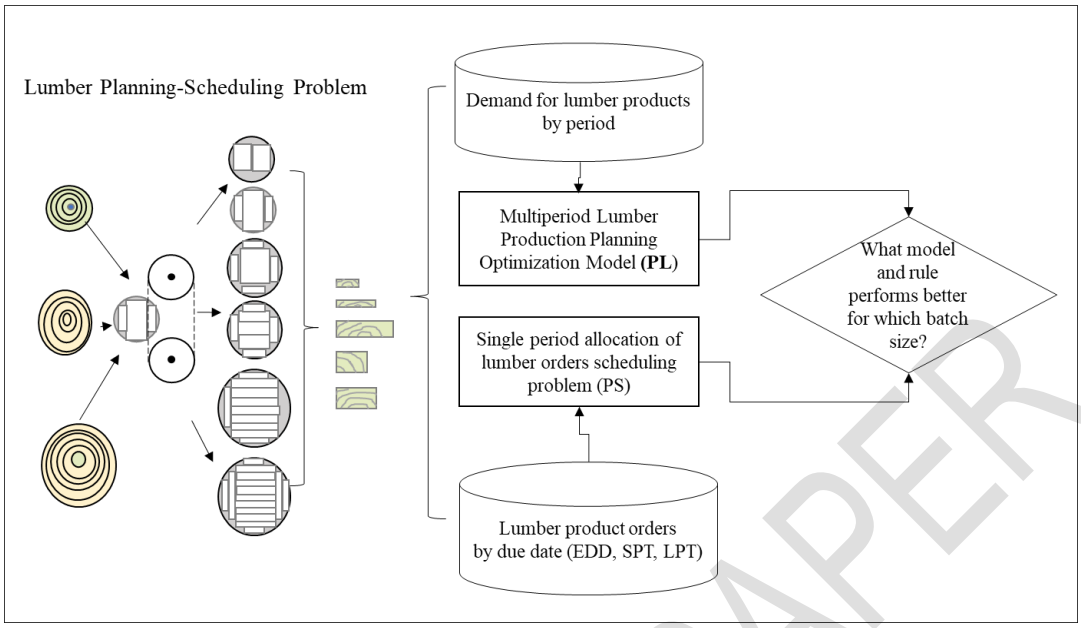


Figure 1: Description of solution approaches for the lumber production problem

When testing the planning and scheduling approaches (i.e., PL and PS models) with conflicting objectives, not all the model runs were feasible for the lumber product demand and order size scenarios used. Consequently, both model formulations were adjusted to accommodate backlogs in the PL model and to accept delayed orders by relaxing the earliness constraint in the PS model. No economic penalties were imposed on either backlogs or delayed orders due to the subjective nature of these values and the absence of satisfactory criteria. However, these delays were factored into determining key model performance metrics (e.g., backlog ratios, earliness, or lateness ratios). In the PL model, the total backlogs divided by the lumber product demands determined a value known as the backlog ratio for sawn lumber and anti-stained lumber (i.e., A_BO1(%), and A_BO2(%) in the model). A lower ratio indicates a better solution, signifying lower backlog volumes relative to demand.

In the PS model, metrics were derived from earliness and lateness of orders. Order earliness or lateness in hours was determined based on the time an order was produced in relation to its due date (i.e., $A1_p$, $A2_p$, $B1_p$, and $B2_p$ in the model). The hours of earliness or lateness for each order relative to its due date were used to calculate the ratio of earliness or lateness (%) (i.e., $Tfea1_p$, $Tfea2_p$, $Tfde1_p$, $Tfde2_p$ in the model). The ratio of earliness or lateness for each order, multiplied by the order's volume, determined the early or late volume of the order (i.e., $Ad\_1\_vol_p$, $Ad\_2\_vol_p$, $Ba\_1\_vol_p$, and $Ba\_2\_vol_p$ in the model). The summation of early and late volumes per order determined the total late or early volumes (i.e., AV1, AV2, BV1, and BV2 in the model). Finally, the summation of the earliness or lateness ratio of orders determined the overall earliness or lateness ratio, equivalent to the backlog ratio of the PL model (i.e., A_AO1, A_AO2, A_BO1, and A_BO2 in the model).

The PL model allows for backlogs that need to be produced immediately in the subsequent planning period (refer to constraints 2 and 3 of model PL in Appendix A). In contrast, the PS model accommodates overdue orders, resulting in delays relative to the due dates (i.e., hours of delay). Technically, these delays were translated into volumes using sawing and anti-stain process productivity, representing equivalent backlogged volumes in the PL model (constraints 40, 41, 50, 51, 54, and 55 of model PS in Appendix B). In the case of PS model, overdue orders and the corresponding backlog volumes did not incur penalties (for detailed formulations of the models, refer to Appendix A and B).

Results and discussion

Analysis of the lumber manufacturing costs

The results were evaluated with consideration of the modeling approach, the applied static heuristic, and the size of lumber orders. For large orders, the lumber manufacturing cost exhibit a change of no more than 0,1 %. In the case of small orders, the lumber cost did not vary beyond 1,7 %, and for mixed orders, the cost did not change by more than 0,5 %. Orders with high variation experienced a cost change of no more than 0,1 %, while orders with low variation saw a cost change of no more than 0,2 % (Table 3).

Table 3: Lumber manufacturing costs for the planning and scheduling approaches.

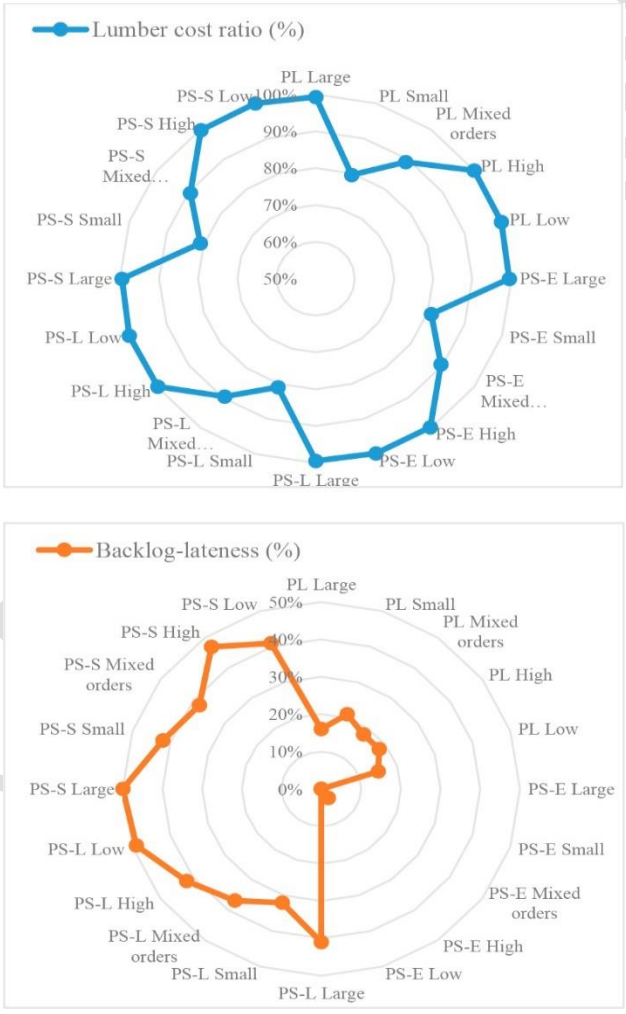
Model	Lumber demand	Backlogs or overdue orders Cost ($)	Difference (%)	Backlog-rate Vol (m^3)	Backlog-lateness Ratio (%)
PL	Large	448498	0 %	1199	16 %
	Small	359539	0 %	1285	21 %
	Mixed orders	402317	0 %	1257	18 %
	High	451417	0 %	1371	18 %
	Low	450759	0 %	1149	15 %
PS-E	Large	448880	0,1 %	0	0 %
	Small	365731	1,7 %	0	0 %
	Mixed orders	404045	0,4 %	0	0 %
	High	450787	0,1 %	3	3 %
	Low	451036	0,1 %	0	0 %
PS-L	Large	449018	0,1 %	3130	41 %
	Small	365760	1,7 %	1980	32 %
	Mixed orders	404326	0,5 %	2555	37 %
	High	450919	0,1 %	3207	42 %
	Low	451552	0,2 %	3832	49 %
PS-S	Large	449034	0,1 %	3891	50 %
	Small	365760	1,7 %	3353	42 %
	Mixed orders	404326	0,5 %	2661	38 %
	High	450919	0,1 %	3973	47 %
	Low	451552	0,2 %	3293	41 %

Analysis of due dates and backlog

The impact of the relaxation on overdue volumes was more pronounced than on lumber manufacturing costs. Larger averages of delayed orders (in hours) and consequently, backlogged volumes, were observed with the PS-S and PS-L in comparison to PS-E. However, the average backlogged volumes for the PL approach were also noteworthy. For instance, the PS-E had an average of backlogged volumes equivalent to 0,5 % of the volumes

of lumber demand ordered. In the case of PL, the average backlogged volumes represented 17 % of the volumes of lumber demand ordered. For PS-L, the average backlogged volumes were 41 % of the volume of orders ordered, and for PS-S, the average backlogged volumes were 44 % of the volume of orders ordered, irrespective of the order size (Table 3). Figure 2 shows the distribution of backlogged volumes across model scenarios.

(a)



(b)

Figure 2: Lumber Manufacturing a) Costs Ratio and b) Backlog lateness ratio.

The PL and PS-E models yielded cost results that were remarkably similar across various order sizes. The PS-E sequence closely resembled how the multi-period PL model handled lumber product demand during the period when the demand was initiated. Consequently, both the PL and PS-E models exhibited very similar costs. However, due to the heuristic nature of the E static heuristic, the PL model consistently generated lower costs. These findings align with the results reported by Maturana *et al.* 2010, where an LP multi-period model and a heuristic scheduling planning tool based on due dates were compared. However, they reported larger differences, such as 52 %, in costs determined by the LP model and the heuristic approach for lumber demand scenarios. This disparity may be attributed to the use of more extensive and specific data perturbations, like increasing the demand of a low-value product by 10 %. In contrast, this research employed random changes within certain rates applied to all lumber demand data. Therefore, the differences reported in this study were smaller; for instance, the PS-E cost was only 1,7 % higher than the PL cost when processing small orders.

As highlighted by Gaudreault *et al.* 2011 for industrial settings, the inevitability of overdue orders is acknowledged. Consequently, the effectiveness of planning approaches can be evaluated through late deliveries. The results revealed that the average ratios of backlogged orders were 0,5% for the PS-E, 17 % for the PL, 41 % for the PS-L, and 44 % for PS-S approaches, irrespective of the order size. These values align with those reported by Gaudreault *et al.* 2011, who observed ratios of backlogged volume of orders ranging between 26 % and 50 % when addressing a similar problem. However, their solution involved utilizing a planning-scheduling MIP and an LPT heuristic model with coordination protocols over an agent-based platform.

The PS-E approach demonstrated intriguing capabilities of both models (i.e., PL model and PS model). It exhibited cost saving when planning orders with high variations in the volume of orders, achieving a 0,2 % lower cost than the PL for the same order sizes. However, the PS-E proved equally efficient as the PL when processing large and small variations orders as well (Table 4). Consequently, the PS-E displayed a greater ability to reduce due date delays and, therefore, overdue volumes compared to the PL.

Furthermore, the PS-L approach could be employed when planning small orders resulting in a cost 0,24 % lower than the average of the PL for the same order size. This potential utilization of PS-L could be extended to these order sizes.

The PL model demonstrated high efficiency in addressing the lumber manufacturing production planning problem as a general lot sizing and scheduling problem with cost minimization (Maness and Adams 1991, Maness and Norton 2002, Singer and Donoso 2007, Vergara *et al.* 2015, Broz *et al.* 2019). LP proved to be a highly effective solution method when adjusting order sizes and testing sequences for order processing. However, this research showed that this model formulation did not ensure the most cost-effective production plan or the lowest backlogged volumes under all circumstances.

While the percentage differences in costs for the model scenarios were small, the economic impact of the PS-E approach could be significant, given the operational scale of this type of facility. Furthermore, the PS-E approach demonstrated the capability to reduce the backlog ratio close to zero, compared to the PL backlog ratio of 17 %. No penalties were imposed to backlogs due to their subjective nature; however, when shortages occur, the demand is either lost or backlogged with substantial costs. In manufacturing, shortage costs are estimated based on lost revenue. Gupta and Starr 2014 suggested a cost ratio of 1:4 between holding inventory cost and backlog cost, while West 1989 assumed that a reasonable value for a

backlogged unit should be the delivery price. Therefore, considering the chosen criterion for penalizing backlogs and keeping in mind its limitations, the PS-E and PL approaches were benchmarked. As a result, if the developed planning method is adopted using a conservative backlog cost, a sawmill producing 300000 m^3 per year would reduce backlogged orders by 51 thousand m^3 (equivalent to a 17 % backlog reduction). Assuming a holding lumber cost of 2 $\$/m^3$, the annual savings would amount to

$408000.

The set-up cost was excluded from lumber processing costs to avoid the incorporation of binary variables for modeling the fixed cost associated with changing production plans, opting instead for an LP formulation. This decision was primarily made to facilitate problem-solving within a reasonable timeframe. The application of this relaxation approach has been observed in previous studies, such as Maturana *et al*. 2010 and Yaghubian *et al.* 2001 in the context of lumber manufacturing production planning and dry kiln scheduling problems. Additionally, the negligible setup times attributed to high-production sawmills, which are now under CNC control for log processing and cant breakdown machines, set-up times are negligible.

Conclusions

The PL approach resulted in the lowest costs, albeit with only a slight advantage compared to the PS-E. However, the PS-E approach demonstrated comparable efficiency to PL but with

lower backlogged volumes. Consequently, the PS-E model is recommended as the best approach for planning lumber production orders in sawmilling operations regardless of lumber order sizes. It is important to note that this conclusion is applicable under specific conditions, including a cut-to-order lumber production environment, a significant emphasis on backlogs valuation or penalization due to overseas lumber customers' focus or specialty lumber products focus, and short-term planning horizons (e.g., Chilean sawmilling industry). Integrated forest companies cater to customers with varying due date requirements, placing a significant emphasis on internal customer with high due date flexibility. Most of the lumber production planning research, however, has been conducted without accounting for any due date flexibility. Typically, only backlogs in terms of the volume of lumber products have been considered. The treatment of overdue orders may vary, contingent on the willingness of customers to accept delays. A promising avenue for new research would involve exploring approaches to model the problem with flexible due dates for customers, as opposed to excessively constraining the lumber manufacturing production planning problem.

The limitations of this research should be acknowledged. Firstly, only 30 model runs were conducted by order size, and an increased number of data sets and model runs would enhance the reliability of the results. Secondly, the data sets were generated based on the ideal lumber demand for this specific sawmill case study, and rates of variation were randomly selected. Consequently, a more in-depth analysis of these rates of variation should be further studied. Lastly, decision-making models for sawmilling and anti-stain operations were formulated, excluding kiln drying operations. Incorporating such operations could prove challenging due to their utilization of parallel resources.

Authorship contributions

F. P. V.: Conceptualization, methodology, software, investigation, data curation, formal analysis, writing-original draft, visualization. C. D. P.: Supervision, validation, writing-review & editing. J. D. N.: Funding acquisition, resources, project administration, writing-review & editing.

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APPENDIX

Appendix: Single nomenclature

Sets

i: Diameter, j: Log grade, k: Sawing pattern, o: Lumber product, from 1…n, n ∈ Z

s: Planning periods p: Orders of lumber products, 1…m, m ∈ Z

Data declaration and Decision Variables:

Logprice_{ij}	:	Log price in diameter i, and log grade j in \$/m^3.
Cilog_s	:	Cost to keep 1 m^3 of log-in inventory in period s.
Cisa_s	:	Cost to keep 1 m^3 of sawn lumber in inventory in period s.
Ciat_s	:	Cost to keep 1 m^3 of anti-stain treated lumber in inventory in period s.
Sawing_cost	:	Cost of sawing in \$ per hour
At_cost	:	Cost of anti-stain treatment in \$ per hour.
Yield_sw_{ijko}	:	Sawmilling yield in [%] of product o, recovered from a log of diameter i, grade j with sawing pattern k.
Yield_at_o	:	Anti-stain treatment yield in [%] of lumber product o.
Cap_sw_s	:	Sawmilling capacity (h) in period s.
Cap_at_s	:	Anti-stain treatment capacity (h) in period s.
Prod_sw_i	:	Sawmilling productivity (h/m^3) when sawing log diameter i.
Prod_at_o	:	Anti-stain treatment productivity (h/m^3) when treating lumber product o.
IOLuSA_o	:	Initial inventory of sawn lumber product o (m^3).
IOLuAT_o	:	Initial inventory of anti-stain treated lumber product o (m^3).
D_at_{os}	:	Demand for anti-stain lumber (m^3) product o, in period s
Cilog	:	Cost in \$ of keep 1 m^3 of log in inventory.
Cisa	:	Cost in \$ of keep 1 m^3 of sawn lumber in inventory.
Ciat	:	Cost in \$ of keep 1 m^3 of anti-stain treated lumber in inventory.
Cap_sw	:	Sawmilling capacity (h).
Cap_at	:	Anti-stain capacity (h).
D_at_{po}	:	Lumber demand (m^3) for anti-stain lumber product o contained in order p.
d_at_p	:	Due date (h) for order p.
U_{ijks}	:	Vol. of logs (m^3) of diameter i, log grade j, sawn with sawing pattern k, in periods.
ISA_{os}	:	Sawn lumber inventory (m^3) of lumber product o, in period s.
IAT_{os}	:	Anti-stained lumber inventory (m^3) of product o, in period s.
V_sa_{os}	:	Sawmill lumber production (m^3) of product o, produced in period s.
W_at_{os}	:	Vol. (m^3) of lumber product o, transferred to anti-strain treatment in period s.

V_at_{os}	:	Anti-stain treatment production (m^3) of lumber product o, in period s.
B_gr_{os}	:	Backlog volume of sawn lumber products o, in period s (m^3).
B_at_{os}	:	Backlog volume of anti-stain treated lumber products o, in period s (m^3).
$A1_p$	:	Advanced time (h) of sawn lumber order p.
$B1_p$	:	Backlogged time (h) of sawn lumber order p.
$A2_p$	:	Advanced time (h) of anti-stain lumber order p.
$B2_p$	:	Backlogged time (h) of anti-stain lumber order p.
$Ba\_1\_vol_p$	:	Delayed volumes produced of the sawn lumber order p (m^3).
$Ba\_2\_vol_p$	:	Delayed volumes produced of the anti-stain lumber order p (m^3).
$Tfde1_p$	:	Time fraction of delay of sawn lumber order p (%).
$Tfea1_p$	:	Time fraction of earliness of sawn lumber order p (%).
$Tfea2_p$	:	Time fraction of earliness of anti-stain lumber order p (%).
$Tfde2_p$	:	Time fraction of delay of anti-stain lumber order p (%).
U_{ijkp}	:	Vol. of logs (m^3) of diameter i, log grade j, sawn with sawing pattern k, to satisfy order p.
$LogV_p$	:	Vol. of logs consumed to satisfy order p (m^3).
V_sa_{po}	:	Sawmill lumber production (m^3) of product o, produced for order p.
ISA_{op}	:	Sawn lumber inventory (m^3) of lumber product o, for order p.
W_at_{po}	:	Vol. of sawn lumber (m^3) product o, transferred to anti-strain process for order p.
V_at_{po}	:	Anti-stain treatment production (m^3) of lumber product o, for order p.
$Sawing\_time_p$	:	Sawing time (h) expend to process order p.
Ad_1_p	:	Ratio of earliness for sawn lumber order p about its due date (%).
Ad_2_p	:	Ratio of earliness for anti-stain lumber order p about its due date (%).
Ba_1_p	:	Ratio of delay for sawn lumber order p about its due date (%).
Ba_2_p	:	Ratio of delay for anti-stain lumber order p about its due date (%).
$Ad\_1\_vol_p$	:	Vol. produced in advance of the due date of the sawn lumber order p (m^3).
$Ad\_2\_vol_p$	:	Vol. produced in advance of the due date of the anti-stain lumber order p (m^3).
OV_p	:	Vol. of anti-stain lumber containing order p.
Where		
At_time_p	:	Time expended to process order p on the anti-stain process (h).
A_A01	:	The summation of earliness ratios of sawn lumber orders (%)
A_A02	:	The summation of earliness ratios of anti-stain lumber orders (%)
A_B01	:	The summation of lateness ratios of sawn lumber orders (%)
A_B02	:	The summation of lateness ratios of anti-stain lumber orders (%)
AV1/AV2	:	The summation of early vol. of sawn lumber/ anti-stain lumber orders (m^3).
BV1/BV2	:	The summation of late vol. of sawn lumber/ anti-stain lumber orders (m^3).

Appendix A: Plan (PL) model

Objective function

$$Min : \sum_{ijks} U_{ijks} \times Logprice_{ij} + \sum_{os} IAT_{os} \times Ciat_s + \sum_{ijks} U_{ijks} \times Prod\_sw_i \times \text{Sawing\_cost} + \sum_s \text{W\_at}_{\text{os}} \times \text{Prod\_at}_o \times At\_cost \quad (1)$$

Subject to:

$$\sum_{ijks} U_{ijks} \times Yiel\_sw_{ijkos} = V\_sa_{os} \quad \forall o, s = 1 \quad (2)$$

$$\sum_{ijks} U_{ijks} \times Yiel\_sw_{ijkos} = V\_sa_{os} + B\_gr_{os-1} \quad \forall o, s > 1 \quad (3)$$

$$\sum_{ijks} U_{ijks} \times Prod\_sw_i \leq Cap\_saw_s \quad \forall s \quad (4)$$

$$V\_sa_{os} - ISA_{oj} - W\_at_{os} = 0 \quad \forall o, s = 1 \quad (5)$$

$$V\_sa_{os} + ISA_{os-1} - ISA_{os} + B\_gr_{os} - W\_at_{os} = 0 \quad \forall o, s > 1 \quad (6)$$

$$W\_at_{os} \times Yield\_at_o = V\_at_{os} \quad \forall o, s = 1 \quad (7)$$

$$W\_at_{os} \times Yield\_at_o = V\_at_{os} + B\_at_{os-1} \quad \forall o, s > 1 \tag{8}$$
$$V\_at_{os} + IAT_{os-1} = D\_at_{os} \quad \forall o, s = 1 \tag{9}$$
$$V\_at_{os} + IAT_{os-1} - IAT_{os} + B\_at_{os} = D\_at_{os} \quad \forall o, s > 1 \tag{10}$$
$$\sum_{os} W\_at_{os} \times Prod\_at_o \leq Cap\_at_s \quad \forall s \tag{11}$$
$$B\_gr_{os} = 0 \quad \forall o, s = 6 \quad \text{and} \quad B\_at_{os} = 0 \quad \forall o, s = 6 \tag{12}$$
$$ISA_{os} = 0 \quad \forall o, s = 6 \tag{13}$$
$$ITA_{os} = 0 \quad \forall o, s = 6 \tag{14}$$
$$\text{Sawing\_time}_i = \sum_{ijks} U_{ijks} \times \text{Prod\_sw}_i \quad \forall i \tag{15}$$
$$\text{At\_time}_o = \sum_s \text{W\_at}_{os} \times \text{Prod\_at}_o \quad \forall \text{s} \tag{16}$$
$$Log\ cost: Log\_cost = \sum_{ijks} Uijks \times Logprice_{ij} \tag{17}$$
$$Log\ input: Log\_input = \sum_{ijks} U_{ijks} \tag{18}$$
$$Sawing\ cost: Sawing\_cost = \sum_i Sawing\_time_i \times Sawing\_cost \tag{19}$$
$$Anti\text{-}stain\ treatment\ cost: At\_cost = \sum_o At\_time_o \times At\_cost \tag{20}$$
$$Cost\ of\ holding\ sawn\ lumber\ in\ inventory: Lu\_gr\_icost = \sum_{os} ISA_{os} \times Cisa_s \tag{21}$$
$$Cost\ of\ holding\ antistain\ lumber\ in\ inventory: Lu\_at\_icost = \sum_{os} IAT_{os} \times Ciat_s \tag{22}$$
$$Green\ lumber\ production: GRVLumber = \sum_{os} V\_sa_{os} \tag{23}$$
$$Anti\text{-}stain\ lumber\ production: ATLumber = \sum_{os} V\_at_{os} \tag{24}$$
$$Green\ lumber\ demand: AT\_Demand = \sum_{os} D\_at_{os} \tag{25}$$
$$Backlogged\ volume\ of\ green\ lumber: BV1 = \sum_{os} B\_gr_{os} \tag{26}$$
$$Backlogged\ volume\ of\ anti\text{-}stain\ lumber: BV2 = \sum_{os} B\_at_{os} \tag{27}$$

The objective function (1) minimizes the lumber manufacturing costs, which are: log costs, sawn and anti-stain treated lumber inventory costs, and sawing and anti-stain processing costs. Backlogs were allowed, consequently, several constraints, and metrics to account for backlog volumes. The model constraints are constraints (2) and (3), which ensure that the production of sawn lumber products for period 1, and for periods >1 includes backlogs from the previous period. The sawmilling capacity constraint (4), the summation of the time expended to saw logs, cannot exceed sawing capacity per period; the flow balance constraint (5) for period 1, the sawn lumber production less the period inventory is transferred to the anti-stain process. The flow balance constraint (6) for periods >1, ensures that the flow of sawn lumber products from the sawmill to the anti-stain treatment process includes the backlogs. The anti-stain treated production for period 1 (7), where sawn lumber products are processed with anti-stain yields to produce anti-stain lumber products; constraint (8) ensures that the production of anti-stain lumber for periods >1 includes backlogs from the previous period; constraint (9) is the market constraint for period 1; constraint (10) ensures that the

demand of anti-stain lumber products for periods >1 includes inventory variation and backlogs; constraint (11) is anti-stain process capacity constraint; constraint (12), (13), and (14) ensures no backlogs, sawn lumber products inventory and anti-stain lumber products inventory for the final period; constraints (15) and (16) determine the sawing time, and anti-stain process time; constraints (17) to (27) are model metrics, finally, all variables must be non-negative.

Appendix B: Plan-sched (PS) model

The PS model does not work with lumber products demand by periods. Instead, it works with orders containing a set of lumber, which should be satisfied on certain due dates. However, as the objective of this research was to test S, E, and L static heuristic schedules, the model solves the problem with only these predetermined schedules. The PS model accepts overdue orders, which means delays in hours of a certain order relative to its due date. Accordingly, orders must be processed with a sequence, which was the E, L, and S static heuristic schedules.

Variables, constraints, and metrics were added to account for backlogged volumes due to orders delays. Delays were transformed in volumes by using sawing and anti-stain process productivity, which enabled to compare backlogged volumes between the formulation of models PL and PS. In such a formulations overdue orders and their equivalent backlogs volumes were not penalized.

For a better understanding, the production of sawn lumber did not have an explicit demand or due date but was driven by the demand and delivery date of the final product.

Consequently, the metrics were always based on the production of stain lumber wood products. The PS formulation follows:

Objective function

$$Min{:}\sum_{ijkp} Uijkp \times Logpricet_{ij} + \sum_p GrV_p \times Tfea1_p \times Cisa + \sum_p GrV_p \times Tfea2_p \times Ciat + \sum_p Sawing\_time_p \times Sawing\_cost + \sum_p \text{At\_time}_p \times At\_cost \quad (30)$$

Subject to:

$$\sum_{ijkp} U_{i,j,k,p} \times Yield\_sw_{i,j,k,o} = V\_sa_{p,o} \quad \forall p, o \quad (31)$$

$$\sum_{ijkp} U_{i,j,k,p} \times Prod\_sw_i - Cap\_sw \leq 0 \quad \forall \text{p} \quad (32)$$

$$\sum_{ijkl} U_{i,j,k,l} \times \text{Prod\_sw}_i + \text{A1}_p - \text{B1}_p - \text{d\_at}_p \leq 0 \quad \forall \text{p}, \text{l} = 1 \ldots \text{p} \quad (33)$$

$$\text{V\_sa}_{p,o} - \text{IOLuSA}_{p,o} - \text{W\_at}_{p,o} = 0 \quad \forall \text{p}, \text{o} \quad (34)$$

$$\text{W\_at}_{p,o} \times \text{Yield\_at}_o - \text{V\_at}_{p,o} = 0 \quad \forall \text{ p}, \text{o} \quad (35)$$

$$\sum_{lo} W\_at_{l,o} \times \text{Prod\_at}_o + \text{A2}_p - \text{B2}_p - \text{d\_at}_p \leq 0 \quad \forall \text{ l} = 1 \ldots \text{p}, \text{o} \quad (36)$$

$$\text{V\_at}_{po} + \text{IOLuAT}_{p,o} - \text{D\_at}_{\text{p,o}} = 0 \quad \forall \text{ p}, \text{o} \quad (37)$$

$$\text{Tfea1}_p = \text{A1}_p/\text{d\_at}_p \quad \forall \text{ p} \quad (38)$$

$$\text{Tfea2}_p = \text{A2}_p/\text{d\_at}_p \quad \forall \text{ p} \quad (39)$$

$$\text{Tfde1}_p = \text{B1}_p/\text{d\_at}_p \quad \forall \text{ p} \quad (40)$$

$$\text{Tfde2}_p = \text{B2}_p/\text{d\_at}_p \quad \forall \text{ p} \quad (41)$$

$$OV_p = \sum_o D\_at_{p,o} \quad \forall \text{ p} \quad (42)$$

$$\sum_{po} W\_at_{p,o} \times Prod\_at_o = Cap\_at \quad \forall \text{p} \quad (43)$$

$$LogV_p = \sum_{ijkp} U_{i,j,k,p} \quad \forall \text{p} \quad (44)$$

$$\text{GrV}_p = \sum_{po} \text{D\_at}_{\text{po}} \quad \forall \text{p} \quad (45)$$

$$\text{Sawing\_time}_p = \sum_{ijkp} U_{i,j,k} \times \text{Prod\_sw}_i \quad \forall \text{p} \quad (46)$$

$$\text{At\_time}_p = \sum_o W\_at_{po} \times \text{Prod\_at}_o \quad \forall \text{p} \quad (47)$$

$$\text{Ad\_1}_p = \text{A1}_p/\text{d\_at}_p \times 100\% \quad \forall \text{p} \quad (48)$$

$$\text{Ad\_2}_p = \text{A2}_p/\text{d\_at}_p \times 100\% \quad \forall \text{p} \quad (49)$$

$$\text{Ba\_1}_p = \text{B1}_p/\text{d\_at}_p \times 100\% \quad \forall \text{p} \quad (50)$$

$$\text{Ba\_2}_p = \text{B2}_p/\text{d\_at}_p \times 100\% \quad \forall \text{p} \quad (51)$$

$$\text{Ad\_1\_vol}_p = \text{Ad\_1}_p \times \text{GrV}_p/100\% \quad \forall \text{p} \quad (52)$$

$$\text{Ad\_2\_vol}_p = \text{Ad\_2}_p \times \text{GrV}_p/100\% \quad \forall \text{p} \quad (53)$$

$$\text{Ba\_1\_vol}_p = \text{Ba\_1}_{\text{p}} \times \text{GrV}_p/100\% \quad \forall \text{p} \quad (54)$$

$$\text{Ba\_2\_vol}_p = \text{Ba\_2}_p \times \text{GrV}_p/100\% \quad \forall \text{p} \quad (55)$$

Where:

$$Log\ cost{:}\ \text{Log\_cost} = \sum_{ijkp} Uijkp \times Logpricet_{ij} \quad (56)$$

$$Log\ input{:}\ Log\_input = \sum_{ijkp} U_{ijkp} \quad (57)$$

$$Sawing\ cost{:}\ Sa\_cost = \sum_p Sawing\_time_p \times Sawing\_cost \quad (58)$$

$$Anti{-}stain\ treatment\ cost{:}\ At\_cost = \sum_p At\_time_p \times At\_cost \quad (59)$$

$$Cost\ of\ holding\ sawn\ lumber\ in\ inventory: Lu\_sa\_icost = \sum_p GrV_p \times Tfea1_p \times Cisa \quad (60)$$

$$Cost\ of\ holding\ antistain\ lumber\ in\ invent: Lu\_at\_icost = \sum_p GrV_p \times Tfea2_p \times Ciat \quad (61)$$

$$Sawn\ lumber\ production{:}\ GRLumber = \sum_{po} V\_sa_{po} \quad (62)$$

$$Total\ volume\ of\ backlogged\ sawn\ lumber\ orders{:}\ GRBacklog = \sum_{po} B\_gr_{po} \quad (63)$$

$$Anti{-}stain\ lumber\ production{:}\ ATLumber = \sum_{po} V\_at_{po} \quad (64)$$

$$Total\ volume\ of\ backlogged\ anti{-}stain\ lumber\ orders: ATBacklog = \sum_{po} B\_at_{po} \quad (65)$$

$$Anti{-}stain\ lumber\ demand{:}\ AT\_Demand = \sum_{po} D\_at_{po} \quad (66)$$

$Average\ ratio\ of\ earliness\ for\ sawn\ lumber$: $A\_A01 = \sum_p Ad1_p\ /6$ (67)
$Average\ ratio\ of\ earliness\ for\ anti{-}stain\ lumber$: $A\_AO2 = \sum_p Ad2_p\ /6$ (68)
$Average\ ratio\ of\ delay\ for\ sawn\ lumber$: $A\_B01 = \sum_p Ba1_p\ /6$ (69)
$Average\ ratio\ of\ delay\ for\ anti{-}stain\ lumber$: $A\_BO2 = \sum_p Ba2_p\ /6$ (70)
$Sawn\ lumber\ produced\ in\ advance$: $AV1 = \sum_p Ad\_1\_vol_p$ (71)
$Anti\ stain\ lumber\ produced\ in\ advance$: $AV2 = \sum_p Ad\_2\_vol_p$ (72)
$Sawn\ lumber\ produced\ in\ delay$: $BV1 = \sum_p Ba\_1\_vol_p$ (73)
$Anti\ stain\ lumber\ produced\ in\ delay$: $BV2 = \sum_p Ba\_2\_vol_p$ (74)
GrV_p: $Volume\ sawn\ lumber\ produced\ to\ order\ p$ (75)

The objective function (30) minimizes lumber manufacturing costs, which are log costs, sawn lumber, and anti-stain inventory costs, plus sawing and anti-stain treatment costs. Additionally, a set of constraints were applied which were the following:

The sawn lumber production constraint (31), where logs of a certain diameter, and grade, are processed with a certain sawing pattern to produce certain lumber product o, to satisfy order p; the sawmilling capacity constraint (32), the summation of time to process logs of all orders cannot exceed sawing capacity in hours. Surplus and slack variables were added to the earliness constraint to make the solution feasible, the earliness constraint (33), ensures that the processing time of sawn lumber order p, and processing times of preceding orders can be lower or exceed the due date of order p. Thus, the surplus or slack variables capture the delay or earliness of each order. The flow balance equation (34), where the sawn wood production plus the sawn lumber inventory is transferred to the anti-stain process. The anti-stain treated production constraint (35), where sawn lumber is processed with certain anti-stain yields to produce certain anti-stain lumber products to satisfy order p. The earliness constraint (36) for anti-stain lumber orders, which ensures that the processing time of order p, and processing times of preceding orders must be ≤ to the due date of order p. Thus, the surplus or slack

variables capture the delay or earliness of each order. The market constraint (37), which ensures that the anti-stain lumber production plus the anti-stain inventory must be equal to the anti-stain lumber demand for all orders of anti-stain products.

Additionally, constraint (38) determines the time fraction of earliness of sawn order p (%) as the ratio between the earliness of order p (h), and its due date (h). Constraint (39) determines the time fraction of earliness of anti-stain lumber order p (%) as the ratio between the earliness of order p (h), and its due date (h). Constraint (40) determines the time fraction of delay of sawn lumber order p (h), and its due date (h). Constraint (41) determines the time fraction of delay of anti-stain lumber order p (h), and its due date (h). Constraint (42) determines the volume (in m^3) of each anti-stain lumber product order. Constraint (43) ensures that the summation of the time expended to process sawn lumber orders does not exceed anti-stain process capacity. Constraint (44) determines the volume of logs consumed (m^3) for order p. Constraint (45) determines the volume (m^3) of order p of anti-stain lumber products. Constraint (46) determines the sawing processing time of order p (h). Constraint (47) determines the anti-stain processing time of order p (h). Constraint (48) determines the ratio of earliness of order p of sawn lumber (%) in relation to its due date. Constraint (49) determines the ratio of earliness of order p of anti-stain lumber (%) in relation to its due date. Constraint (50) determines the ratio of delay of order p of sawn lumber (%) in relation to its due date. Constraint (51) determines the ratio of delay of order p of anti-stain lumber (%) in relation to its due date. Constraint (52) determines the volume of sawn lumber (m^3) produced in advance of order p. Constraint (53) determines the volume of anti-stain lumber (m^3) produced in advance of order p. Constraint (54) determines the backlog volume of sawn lumber (m^3) produced for order p. Constraint (55) determines the backlog volume of anti-

stain lumber (m^3) produced for order p. Equations (56) to (75) are model metrics and manufacturing costs. Finally, all decision variables must be non-negative.